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**A NEW METHOD FOR POPULATION FORECASTING BASED ON FUZZY TIME  
SERIES WITH HIGHER FORECAST ACCURACY RATE**

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**ABSTRACT**

The Time-Series models have been used to make predictions in whether forecasting, academic enrollments, etc. Song & Chissom introduced the concept of fuzzy time series in 1993. Over the past 19 years, many fuzzy time series methods have been proposed for population forecasting. But the forecasting accuracy rate of the existing methods is not good enough. These methods have either used actual population or difference of population as the universe of discourse. And either used frequency density based partitioning or nature- ratio based partitioning. In this paper, we proposed a method based on fuzzy time series, which gives the higher forecasting accuracy rate than the existing methods. The proposed method used the percentage change as the universe of discourse and mean based partitioning. To illustrate the forecasting process, the historical population of Azerbaijan is used.

**KEYWORDS:** fuzzy set, fuzzy time series, time variant model, first order model, forecast error.

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**INTRODUCTION**

Forecasting plays an important role in our day- to- day life. If there is an uncertainty about the future, decision makers need to forecast. Forecasting is the process of predicting future outcomes, by which decision makers analyze the related data and graphs to decide and take the best decisions for the future. The time series forecasting problem has attracted more attention for at least two reasons. First, most business, economic, and financial data are time series. Second, the technology for making and evaluating time series forecast is well developed. During last few decades, various approaches have been developed for time series forecasting, but the classical time series methods can not deal with the forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets [5]. To overcome this drawback, Song & Chissom presented the theory of fuzzy time series in 1993[13]. Fuzzy time series model deal with the both linguistic & numerical values.

Fuzzy forecasting methods have been used to model enrollment data for the University of Alabama ([1] - [9], [11] - [13], [18]-[19]), car fatalities ([14] - [15]), food grain production [16], and population forecasting [17]. Hwang, Chen & Lee [2] and Sah & Degtiarev[6] used the differences of the enrollments and Stevenson & Porter [8] used the percentage change of year- to- year enrollments of the University of Alabama as the universe of discourse. For more than one decade, different fuzzy time series models have also been applied to solve various domain problems, such as financial forecasting, temperature forecasting, etc. Fuzzy time series models also used for forecasting ICT products. The objective of the present work is to build, implement and test the fuzzy time series method for population forecasting, and comparison of the result with existing forecasting methods.

In section 2, we briefly review the basic concepts and definitions of fuzzy time series. In section 3, we proposed a new method by using percentage change as the universe of discourse and mean based partitioning. The proposed method has been implemented on the historical population of Azerbaijan. In section 4, we compared the forecasting result of proposed method with the forecasting results of the existing methods. Finally the concluding remarks are discussed in section 5.

**CONCEPT OF FUZZY TIME SERIES****Definition 1: Fuzzy Set**

Fuzzy sets are sets whose elements have degree of membership.

Let  $U$  be the universe of discourse,  $U = \{u_1, u_2, \dots, u_n\}$ , and let  $A$  be a fuzzy set in the universe of discourse  $U$  defined as follows:

$$A = f_A(u_1) / u_1 + f_A(u_2) / u_2 + \dots + f_A(u_n) / u_n,$$

Where  $f_A$  is the membership function of  $A$ ,  $f_A: U \rightarrow [0, 1]$ ,  $f_A(u_i)$  indicates the grade of membership of  $u_i$  in the fuzzy set  $A$ ,  $f_A(u_i) \in [0, 1]$ , and  $1 \leq i \leq n$ , [5].

**Definition 2: Time Series**

A time series is a sequence of data points, measured typically at successive time spaced at uniform time intervals.

**Definition 3: Fuzzy Time Series**

Chronological sequences of imprecise data are considered as time series with fuzzy data. A time series with fuzzy data is referred to as fuzzy time series. [7]. Let  $X(t)$  ( $t = \dots, 0, 1, 2, \dots$ ) be the universe of discourse and be a subset of  $R$ , and let fuzzy set  $f_i(t)$  ( $i = 1, 2, \dots$ ) be defined in  $X(t)$ . Let  $F(t)$  be a collection of  $f_i(t)$  ( $i = 1, 2, \dots$ ). Then,  $F(t)$  is called a fuzzy time series of  $X(t)$  ( $t = \dots, 0, 1, 2, \dots$ ) [5].

**Definition 4: Time variant and Time-invariant fuzzy time series**

Let  $F(t)$  be a fuzzy time series and let  $R(t, t - 1)$  be a first-order model of  $F(t)$ . If  $R(t, t - 1) = R(t - 1, t - 2)$  for any time  $t$ , then  $F(t)$  is called a time-invariant fuzzy time series.

If  $R(t, t - 1)$  is dependent on time  $t$ , that is,  $R(t, t - 1)$  may be different from  $R(t - 1, t - 2)$  for any  $t$ , then  $F(t)$  is called a time-variant fuzzy time series, [5].

**Definition 5: First order model**

If  $F(t)$  is caused by  $F(t - 1)$ , denoted by  $F(t - 1) \rightarrow F(t)$ , then this relationship can be represented by  $F(t) = F(t - 1) \circ R(t, t - 1)$ , where the symbol " $\circ$ " denotes the Max-Min composition operator;  $R(t, t - 1)$  is a fuzzy relation between  $F(t)$  and  $F(t - 1)$  and is called the first-order model of  $F(t)$ , [5].

**Definition 6: Forecast Error**

A forecast error is the difference between the actual or real and the predicted or forecast value of a time series.

$$\text{Error} = \text{Actual value} - \text{forecasted value}$$

**PROPOSED METHOD**

In this section, we proposed a new method by using percentage change as the universe of discourse and mean based partitioning. The aim of this study is to propose a method that is aimed to attain better forecasting accuracy by using fuzzy time series. It should be emphasized that for forecast it uses only historical data in the numerical form (actual population) without any additional pieces of knowledge. The historical populations of Azerbaijan are shown in Table I [17].

Finally, step- by- step forecasting process looks as follows:

Step 1: Define the universe of discourse (universal set  $U$ ). Partition  $U$  into equally length intervals.

Step 2: Define fuzzy sets  $X_i$ , and fuzzify actual historical data.

Step 3: Forecast and defuzzify the forecasted output.

**TABLE.1**  
**THE HISTORICAL POPULATION OF AZERBAIJAN [17]**

Year	Population
1988	6928.0
1989	7021.2
1990	7131.9
1991	7218.5
1992	7324.1
1993	7440.0
1994	7549.6
1995	7643.5
1996	7726.2
1997	7799.8
1998	7879.7
1999	7953.4
2000	8016.2
2001	8081.0

Step 1: Define the universe of discourse  $U$  and partition it into intervals of equal length. The percentage change of enrollments from year to year is given in Table II and ranges from 0.79% to 1.58%. For example, assume that the universe of discourse  $U = [0.7, 1.7]$  is partitioned into five equal intervals.

**TABLE.2**  
**THE YEAR- TO- YEAR PERCENTAGE CHANGE OF POPULATION**

Year to Year	Change
1988-89	1.35%
1989-90	1.58%
1990-91	1.21%
1991-92	1.46%
1992-93	1.58%
1993-94	1.47%
1994-95	1.24%
1995-96	1.08%
1996-97	0.95%
1997-98	1.02%
1998-99	0.95%
1999-2000	0.79%
2000-2001	0.81%

Step 2: Get a mean of the original data. Get the means of frequency of each interval shown in Table III. Compare the means of original and frequency of each interval and then split the five intervals into number of sub-intervals respectively.

**TABLE.3**  
**MEANS OF ORIGINAL DATA AND FREQUENCY OF INTERVALS DATA**

Interval	Number of Data
[0.7, 0.9]	2
[0.9, 1.1]	4
[1.1, 1.3]	2
[1.3, 1.5]	3
[1.5, 1.7]	2

Step 3: Define each fuzzy set  $X_i$  based on the re-divided intervals and fuzzify the historical data shown in Table I, where fuzzy set  $X_i$ , denotes a linguistic value of the year to year percentage change represented by a fuzzy set.

**TABLE.4**  
**FUZZY INTERVALS USING MEAN BASED PARTITIONING**

Linguistic	Interval
X <sub>1</sub>	[0.7, 0.8]
X <sub>2</sub>	[0.8, 0.9]
X <sub>3</sub>	[0.9, 0.95]
X <sub>4</sub>	[0.95, 1]
X <sub>5</sub>	[1, 1.05]
X <sub>6</sub>	[1.05, 1.1]
X <sub>7</sub>	[1.1, 1.2]
X <sub>8</sub>	[1.2, 1.3]
X <sub>9</sub>	[1.3, 1.367]
X <sub>10</sub>	[1.367, 1.434]
X <sub>11</sub>	[1.434, 1.5]
X <sub>12</sub>	[1.5, 1.6]
X <sub>13</sub>	[1.6, 1.7]

Step 4: Defuzzify the fuzzy data shown in Table V, using the forecasting formula [7].

$$t_j = \begin{cases} \frac{1 + 0.5}{\frac{1}{a_1} + \frac{0.5}{a_2}} & , \text{if } j = 1. \\ \frac{0.5 + 1 + 0.5}{\frac{0.5}{a_{j-1}} + \frac{1}{a_j} + \frac{0.5}{a_{j+1}}} & , \text{if } 2 \leq j \leq n - 2. \\ \frac{0.5 + 1}{\frac{0.5}{a_{n-1}} + \frac{1}{a_n}} & , \text{if } j = n. \end{cases}$$

Where a<sub>j-1</sub>, a<sub>j</sub>, a<sub>j+1</sub> are the midpoints of the fuzzy intervals X<sub>j-1</sub>, X<sub>j</sub>, X<sub>j+1</sub> respectively. t<sub>j</sub> yields the predicted year to year percentage change of population. Use the predicted percentage on the previous year’s population to determine the forecasted population. The forecasted population is given in Table V.

**A COMPARISON OF DIFFERENT FORECASTING METHODS**

As in [8], we use the average forecasting error rate (AFER) and mean square error (MSE) to compare the forecasting results of different forecasting methods shown in Table VI:

$$AFER = (|A_i - F_i| / A_i) / n \times 100\%$$

$$MSE = (\sum_{i=1 \text{ to } n} (A_i - F_i)^2) / n$$

Where A<sub>i</sub> denotes the actual population and F<sub>i</sub> denotes the forecasting population of year i, respectively. In A. M. Abbasov & M. H. Mamedova’s fuzzy time series method AFER comes to 0.13%, where as for proposed fuzzy time series method has 0.0149%. Figure 1 shows the comparison of different forecasting methods.

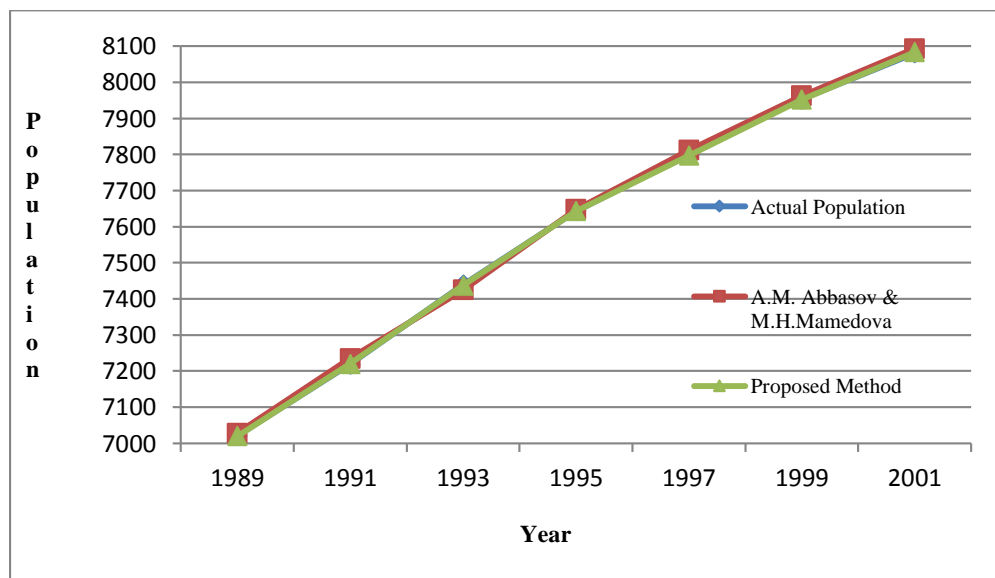
**TABLE.5**  
**FORECASTING RESULT OF THE PROPOSED METHOD**

Year	Population (A <sub>i</sub> )	Percentage	Fuzzy set	Predicted Percentage	Forecast (F <sub>i</sub> )	A <sub>i</sub> -F <sub>i</sub>	(A <sub>i</sub> -F <sub>i</sub> ) <sup>2</sup>	A <sub>i</sub> -F <sub>i</sub>   / A <sub>i</sub>
1988	6928.0	-	-	-	-	-	-	-
1989	7021.2	1.35%	X <sub>9</sub>	1.33%	7020.1	1.1	1.21	0.000156
1990	7131.9	1.58%	X <sub>12</sub>	1.55%	7130.0	1.9	3.61	0.000266
1991	7218.5	1.21%	X <sub>8</sub>	1.24%	7220.3	-1.8	3.24	0.000249
1992	7324.1	1.46%	X <sub>11</sub>	1.47%	7324.6	-0.5	0.25	0.000068
1993	7440.0	1.58%	X <sub>12</sub>	1.55%	7437.6	2.4	5.76	0.000322
1994	7549.6	1.47%	X <sub>11</sub>	1.47%	7549.6	0	0	0

1995	7643.5	1.24%	X <sub>8</sub>	1.24%	7643.5	0	0	0
1996	7726.2	1.08%	X <sub>6</sub>	1.08%	7726.2	0	0	0
1997	7799.8	0.95%	X <sub>3</sub>	0.92%	7797.3	2.5	6.25	0.000320
1998	7879.7	1.02%	X <sub>5</sub>	1.02%	7879.7	0	0	0
1999	7953.4	0.95%	X <sub>3</sub>	0.92%	7952.2	1.2	1.44	0.000150
2000	8016.2	0.79%	X <sub>1</sub>	0.78%	8015.4	0.8	0.64	0.000099
2001	8081.0	0.81%	X <sub>2</sub>	0.84%	8083.5	-2.5	6.25	0.000309
							MSE=2.2038	AFER= 0.0149%

**TABLE.6**  
**COMPARISON OF DIFFERENT FORECASTING METHODS**

Year	Population	A. M. Abbasov & M. H. Mamedova [17]	Proposed Method
1988	6928.0	6926.7	-
1989	7021.2	7028.0	7020.1
1990	7131.9	7114.5	7130.0
1991	7218.5	7224.9	7220.3
1992	7324.1	7308.5	7324.6
1993	7440.0	7425.1	7437.6
1994	7549.6	7544.3	7549.6
1995	7643.5	7647.9	7643.5
1996	7726.2	7736.5	7726.2
1997	7799.8	7812.0	7797.3
1998	7879.7	7884.0	7879.7
1999	7953.4	7962.6	7952.2
2000	8016.2	8034.4	8015.4
2001	8081.0	8093.4	8083.5
AFER	-	0.13%	0.0149%
MSE	-	-	2.2038



**Fig1. Comparison of different forecasting methods**

## CONCLUSION

In this paper, we proposed a new method by using percentage change as the universe of discourse and mean based partitioning. From Table VI, one sees that the proposed method provide the smallest AFER and MSE. For future work, we will develop new method for forecasting data based on different intervals to get a higher forecasting accuracy.

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